



DEPARTMENT OF MATHEMATICS, COMPUTER SCIENCE AND PHYSICS

MATH TOURNAMENT 2023

CIPHERING ROUND

10th February 2023

Q.1. Solve for x :

$$4^{2x^2+1} = 64^{3x-2}$$

Solution: $x = 1, \frac{7}{2}$

Q.2. Expand the following logarithmic function:

$$\log(\sqrt[3]{xyz})$$

Solution:

$$\frac{1}{3} \log x + \frac{1}{3} \log y + \frac{1}{3} \log z \quad \text{or} \quad \frac{\log x + \log y + \log z}{3}$$

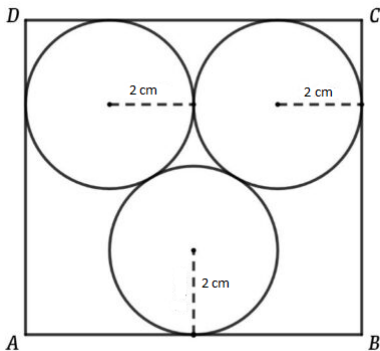
Q.3.

Find the next two terms of the pattern:

100, 101, 97, 106, 90, —, —

Solution: 115, 79

Q.4.



Find the area of the rectangle ABCD if the radius of each circle is 2 cm.

Solution: $8(4 + 2\sqrt{3}) \text{ cm}^2$

Q.5. Simplify the following expression:

$$\frac{4x^4 yz^{-7}}{8x^{10} y^{-4} z^2}$$

Solution:

$$\frac{y^5}{2x^6z^9}$$

Q.6.

If $i = \sqrt{-1}$ is the imaginary unit, then

$$i^{2023} = ?$$

Solution: $-i$ or $-\sqrt{-1}$

Q.7.

The growth in population of a biology experiment fits $A(t) = 445e^{.013t}$, where t is the number years since 1987. Estimate the population in the year 2000.

Solution: 527

Q.8.

Simplify the following expression:

$$\sum_{p=1}^{300} (ab^2p)$$

Solution: $45150ab^2$

Q.9.

Out of the top 10 ciphering students last year, 4 were females. What is the probability that 3 of them were in the top 5? Express your answer in reduced fractional form.

Solution: $\frac{5}{21}$

Q.10.

Average grade of a class of 10 students in Test 1 was 73. One student who had scored 83 was dropped from the class and two new students with the scores of 65 and 47 were added to the class. What is the current average grade of the class in Test 1?

Solution: 69

Q.11. Solve for x :

$$\log(x + 9) + \log(x) = 1$$

Solution: $x = 1$

Q.12.

A group of students consists 4 girls and 3 boys. If we arrange them to sit around the table with the “boy-girl-boy-girl” pattern, how many different arrangements can there be?

Solution: 720